We'll play around with the state functions and the partition functions of different ensembles

We'll often need to swap a thermodynamic variable *x* with another one *s*
That's why we need two mathematical tools:

### *Legendre transform*

$$\tilde{f}(s) = f(x(s)) - sx(s), \qquad s = f'(x)$$

$$f(x) \rightarrow \tilde{f}(s)$$

*e.g. : from the entropy of the microcanonical ensemble to the Helmoltz free energy of the canonical ensemble*

$$S(N, V, E) \rightarrow A(N, V, T)$$

### *Laplace transform*

$$\hat{f}(s) = \int_0^\infty e^{-sx} f(x) dx$$

$$f(x) \rightarrow \hat{f}(s)$$

*e.g. : from the microcanonical partition function to the canonical partition function*

$$\Omega(N, V, E) \rightarrow Q(N, V, T)$$